How Eccentric are LIGO/Virgo Double Neutron Stars?

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We consider the evolution of a double neutron star (DNS), approximated as two point masses in Keplerian orbit, via gravitational radiation reaction to the leading quadrupole order (Peters, 1964):

$$\frac{a(e)}{a_0} = \left(\frac{e}{e_0}\right)^{12/19} \frac{1-e_0^2}{1-e^2} \left(\frac{1+121/304e^2}{1+121/304e_0^2}\right)^{870/2299}, \tag{1}$$

where the evolution in semi-major axis a is parametrised by the eccentricity e, and a_0, e_0 are the semi-major axis and eccentricity at some point of this evolution. For our purposes, we take them to be the semi-major axis and eccentricity at the formation of the DNS, i.e. after the second supernova. We also take a and e to be the respective values when the DNS evolves to LIGO/Virgo-sensitive frequencies.

We can safely take the factors $(1+121/304e^2)^{870/2299}$, $(1+121/304e_0^2)^{870/2299}$, and $1-e^2$ to be unity, such that upon rearrangement to make e the subject,

$$\frac{e}{e_0} \approx \left(\frac{a}{a_0}\right)^{19/12} \frac{1}{(1-e_0^2)^{19/12}}. \tag{2}$$

We find that e/e_0 scales as the ratio of the semi-major axis at detection to the semilatus rectum $a_0(1-e_0^2)$ at formation, to the power of 19/12. We can also recast the equation in terms of the gravitational wave frequency (assumed to be twice the orbital frequency) using $f \propto a^{-3/2}$:

$$\frac{e}{e_0} \approx \left(\frac{f_0}{f}\right)^{19/18} \frac{1}{(1-e_0^2)^{19/12}}. \tag{3}$$

This shows that the ratio of final to initial eccentricity scales approximately linearly with the ratio of initial to final frequency. The DNS eccentricity in the LIGO/Virgo is enhanced by the initial eccentricity e_0 in the factor $e_0/(1-e_0^2)^{19/12}$, which is plotted in Fig. 1 (this is Fig. 1 of Peters (1964)). We see that extreme initial eccentricities are required for a significant enhancement; e.g. for $e_0 < 0.99$, $e_0/(1-e_0^2)^{19/12}$ does not exceed 10^2.

- For DNSs being formed with ~ 1 day periods ($f_0 \sim 10^{-5}$ Hz), observed at a detector lower frequency limit of $f \sim 10$ Hz, $(f_0/f)^{19/18} \sim 10^{-6}$. Therefore if we make the conservative assumption that $e_0 < 0.99$, the eccentricity e at detection decreases by a factor no less than 10^4. For $e_0 < 0.9$, this factor is 10^5.

- If we consider a DNS formed via unstable case BB mass transfer, where the additional common-envelope episode circularises the DNS at birth to $f_0 \sim 10^{-3}$ Hz, this leads to a factor ~ 100 enhancement in e/e_0. In this case, the eccentricity e at detection decreases by at least a factor of 100.

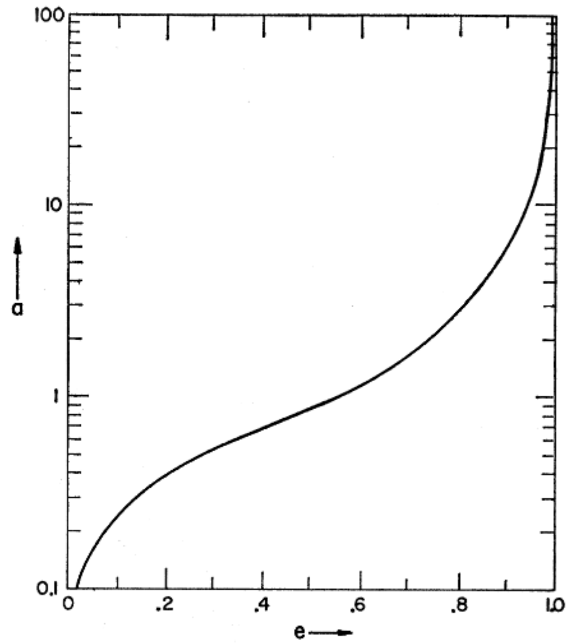

FIG. 1. The semimajor axis a as a function of the eccentricity e in the decay of a two-point mass system. Here, c_0 is chosen to be 1.

Figure 1: a_0 vs. e_0 (Peters, 1964).

References

Peters, P. C. (1964). Gravitational radiation and the motion of two point masses. *Phys. Rev.*, 136:B1224–B1232.